COMMON PRE-BOARD EXAMINATION 2023-24

Subject: MATHEMATICS STANDARD (041)

Class X

MARKING SCHEME

Time: 3 Hrs. Max. Marks: 80

General Instructions:

	SECTION A	
	Section A consists of 20 questions of 1 mark each.	
1.	(c) x^3y^3	1
2.	(d) $10x - 14y = -4$	1
3.	(c) 3	1
4.	(a) $\frac{b}{a}$	1
5.	(d) 98	1
6.	(b) (3,5)	1
7.	(d) infinitely many	1
8.	(c) 5	1
9.	(d) 120^0	1

10.	(a) 3cm	1
11.	(c) 3/2	1
12.	(a) 1/3	1
13.	(c) 60^0	1
14.	(b) 16.8cm	1
15.	(c) $77cm^2$	1
16.	(c) 9	1
17.	(b) 0.69	1
18.	(a) 0.0001	1
19.	(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)	1
20.	(c) Assertion (A) is true but reason (R) is false.	1
	SECTION B	
	Section B consists of 5 questions of 2 marks each.	
21.	Let us assume $5 + 2\sqrt{3}$ is rational, then it must be in the form of p/q where p and q are co-prime integers and $q \neq 0$ i.e $5 + 2\sqrt{3} = p/q$ ------1/2 mark So $\sqrt{3} = \frac{p-5q}{2q}$……………………(i) ------1 mark Since p, q, 5 and 2 are integers and $q \neq 0$, HS of equation (i) is rational. But LHS of (i) is $\sqrt{3}$ which is irrational. This is not possible. This contradiction has arisen due to our wrong assumption that $5 + 2\sqrt{3}$ is rational. So, $5 + 2\sqrt{3}$ is irrational. ------1/2 mark	
22.	It is given that, In ΔABC and DE ∥ BC.	

	So, We know that (using Thales Theorem) $\frac{AD}{DB} = \frac{AE}{CE}$ ------1/2 mark Then, $\frac{AD}{7.2} = \frac{1.8}{5.4}$ $AD = \frac{1.8 \times 7.2}{5.4}$ ------1 mark $AD = \frac{7.2}{3}$ $AD = 2.4$ c. m. ------1/2 mark	
23.	We know that AQ = AR [Tangents from an external point on the circle are equal in length] $\Rightarrow$ AQ = AC + CR But, CR = CP [Tangents from an external point on the circle are equal in length] Therefore, AQ = AC + CP .....(i) Also, AQ = AB + BQ ------1/2 mark But, BQ = BP [Tangents from an external point on the circle are equal in length] $\therefore$ AQ = AB + BP .....(ii) Adding equations (i) and (ii) ------1/2 mark 2AQ = AC + CP + BP + AB ------1/2 mark $\Rightarrow$ 2AQ = Perimeter of $\triangle$ABC $\Rightarrow$ Perimeter of $\triangle$ABC = $2 \times 15 = 30$ cm ------1/2 mark	

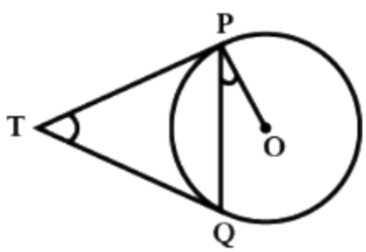
24.	$\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$ $=\frac{2(1+\sin\theta)(1-\sin\theta)}{2(1+\cos\theta)(1-\cos\theta)}$ -------------(1/2m) $=\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ -------------(1/2m) $=\frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$ -------------(1/2m) $=\frac{\cos^2\theta}{\sin^2\theta}$ $=\cot^2\theta=\left(\frac{15}{8}\right)^2=\frac{225}{64}$ -------------(1/2m) **OR** In the given right angled triangle -------------(1/2m) $\tan A=\frac{1}{\sqrt{3}}$ A = 30 Given C = 90 So, "A" + "B" + "C" = "180" -------------(1/2m) So , $B=60^0$ Therefore: sinAcosB+cosAsinB $=\sin30^0\cos60^0+\cos30^0\sin60^0$ $=\frac{1}{2}\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}=\frac{1}{4}+\frac{3}{4}=1$ -------------(1m)	

25.	AOB is a sector of angle 60° of a circle with centre O and radius 17 If AP ⊥ OB and AP = 15 cm Now we have to find the area of the shaded region. In △OPA, ∠O = 90° By using Pythagoras theorem $AO^2 = AP^2 + OP^2$ $17^2 = 15^2 + OP^2$ $OP^2 = 289 - 225$ $OP^2 = 64$ OP = 8 cm ------1/2 mark Area of shaded region = Area of sector AOBA − Area of △OPA $= \frac{x}{360} \times \pi r^2 - \frac{1}{2} \times b \times h$ ------1/2 mark $= \frac{60}{360} \times \frac{22}{7} \times 17 \times 17 - \frac{1}{2} \times 8 \times 15$ ------1/2 mark $= \frac{1}{6} \times \frac{22}{7} \times 289 - 60$ $= 151.38 - 60$ $= 91.38$ cm^2 ------1/2 mark **Hence the area of the shaded region is 91.38 cm^2.** **OR**	

	The figure shows that the tangents drawn from the exterior point to a circle are equal in length. As DR and DS are tangents from exterior point D so, DR = DS---- (1) As AP and AS are tangents from exterior point A so, AP = AS---- (2) As BP and BQ are tangents from exterior point B so, BP = BQ---- (3) ------1/2 mark As CR and CQ are tangents from exterior point C so, CR = CQ---- (4) Adding the equation 1, 2, 3 & 4, we get $DR + AP + BP + CR = DS + AS + BQ + CQ$ ------1/2 mark $(DR + CR) + (AP + BP) = (DS + AS) + (BQ + CQ)$ $CD + AB = DA + BC$ ------1/2 mark $AB + CD = BC + DA$ Hence proved. ------1/2 mark	
	SECTION C	
	Section C consists of 6 questions of 3 marks each	
26.	The Number of room will be minimum if each room accomodates maximum number of participants. Since in each room the same number of participants are to be seated and all of them must be of the same subject. Therefore, the number of participants in each room must be the HCF of 60, 84 and 108. The prime factorisations of 60, 84 and 108 are as under. -----1mark $60 = 2^2 \times 3 \times 5, 84 = 2^2 \times 3 \times 7$ and $108 = 2^2 \times 3^3$ $\therefore$ HCF of 60, 84 and 108 is $2^2 \times 3 = 12$ Therefore, in each room 12 participants can be seated. ----1mark $\therefore$ Number of rooms required $= \dfrac{\text{Total number of participants}}{12}$ $= \dfrac{60 + 84 + 108}{12} = \dfrac{252}{12} = 21$ ------1mark	

27.	For given polynomial x^2 - (k + 6)x +2(2k - 1) Here a = 1, b =-(k = 6), c = 2(2k - 1) ----------(1/2m) Given that; $\therefore$ Sum of zeroes = $\frac{1}{2}$ (product of zeroes) ----------(1/2m) $\Rightarrow \frac{-[-(k+6)]}{1} = \frac{1}{2} \times \frac{2(2k-1)}{1}$ ----------(1m) ⇒ k + 6=2k-1 ⇒ 6+1= 2k - k ----------(1m) ⇒ k = 7 Therefore, the value of k = 7.	

28.	**Let us consider,** One's digit of a two digit number = x and Ten's digit = y ----------(1/2m) So, the number is $x + 10y$ **By interchanging the digits,** One's digit = y and Ten's digit = x ----------(1/2m) Number is $y + 10x$ **As per the statement,** ----------(1/2m) $x + y = 12$ .... (1) ----------(1/2m) $y + 10x = x + 10y + 18$ $y + 10x - x - 10y = 18$ $x - y = 2$ ...(2) **Adding (1) and (2), we have** $2x = 14$ or $x = 7$ ----------(1/2m) **On subtracting (1) from (2),** $2y = 10$ or $y = 5$ ----------(1/2m) **Answer:** **Number = 7 + 10 x 5 = 57** **OR**	

	For x + 2y = 3 or x + 2y - 3 = 0 $a_1 = 1, b_1 = 2, c_1 = -3$ ---------(1/2m) For $(k-1)x + (k+1)y = (k+2)$ or $(k-1)x + (k+1)y - (k+2) = 0$ $a_2 = (k-1), b_2 = (k+1), c_2 = -(k+2)$ ---------(1/2m) For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ ---------(1m) $\Rightarrow \quad \frac{1}{k-1} = \frac{2}{k+1} \neq \frac{3}{k+2}$ and $\quad \frac{1}{k-1} = \frac{2}{k+1}$ ---------(1/2m) $\Rightarrow \quad k+1 = 2k-2$ $\Rightarrow \quad 3 = k$ $\therefore \quad k = 3$ ---------(1/2m)	
29.	 We know that length of tangents drawn from an external point to a circle are equal ∴ TP = TQ ---**(1)** ∴ ∠TQP =∠TPQ (angles of equal sides are equal) ---**(2)** --½ m Now, PT is tangent and OP is radius. ∴ OP⊥TP (Tangent at any point pf circle is perpendicular to the radius through point of cant act) ∴ ∠OPT = 90° --½ m or, ∠OPQ + ∠TPQ = 90° --½ m or, ∠TPQ = 90° −∠OPQ ---**(3)** --½ m	

	In $\triangle PTQ$ $\angle TPQ + \angle PQT + \angle QTP = 180°$ ($\therefore$ Sum of angles triangle is 180°) or, $90° - \angle OPQ + \angle TPQ + \angle QTP = 180°$ or, $2(90° - \angle OPQ) + \angle QTP = 180°$ [from (2) and (3)] or, $180° - 2\angle OPQ + \angle PTQ = 180°$ $\therefore \angle PTQ = 2\angle OPQ$ **OR** Joint OT. Let it meet PQ at the point R. Then ΔTPQ is isosceles and TO is the angle bisector of $\angle PTO$. [$\because$ TP = TQ = Tangents from T upon the circle] $\therefore OT \perp PQ$ $\therefore$ OT bisects PQ.	 --½ m --½ m --½ m

	$PR = RQ = 4$ cm Now, $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2} = 3$ cm Now, $\angle TPR + \angle RPO = 90^\circ (\because TPO = 90^\circ)$ $= \angle TPR + \angle PTR (\because TRP = 90^\circ)$ $\therefore \angle RPO = \angle PTR$ $\therefore$ Right triangle TRP is similar to the right triangle PRO. [By A-A Rule of similar triangles] $\therefore \frac{TP}{PO} = \frac{RP}{RO} \Rightarrow \frac{TP}{5} = \frac{4}{3}$	--½ m --½ m --½ m --½ m --½ m
30.	LHS : $\frac{\sin^3\theta/\cos^3\theta}{1+\sin^2\theta/\cos^2\theta} + \frac{\cos^3\theta/\sin^3\theta}{1+\cos^2\theta/\sin^2\theta}$ $= \frac{\sin^3\theta/\cos^3\theta}{(\cos^2\theta+\sin^2\theta)/\cos^2\theta} + \frac{\cos^3\theta/\sin^3\theta}{(\sin^2\theta+\cos^2\theta)/\sin^2\theta}$ $= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta}$ $= \frac{\sin^4\theta + \cos^4\theta}{\cos\theta\sin\theta}$ $= \frac{(\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$ $= \frac{1 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$ $= \frac{1}{\cos\theta\sin\theta} - \frac{2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$ $= \sec\theta\operatorname{cosec}\theta - 2\sin\theta\cos\theta$ $= $ RHS	½ ½ ½ ½ ½ ½

31.

Class Interval Height (in cm)	Frequency f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
50-75	5	62.5	-5	-25
75-100	6	87.5	-4	-24
100-125	3	112.5	-3	-9
125-150	4	137.5	-2	-8
150-175	3	162.5	-1	-3
175-200	7		0	0
200-225	5	212.5	1	5
225-250	4	237.5	2	8
250-275	8	262.5	3	24
275-300	5	287.5	4	20
	$\sum f_i = 50$			$\sum f_i y_i = -12$

---------table 3 marks

Here, $\sum f_i u_i = -12$; $\sum f_i = 50$, $h = 25$

Mean $M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$ Formula-----1/2 mark

$= 187.5 + \frac{-12}{50} \times 25$

$= 187.5 - 6 = 181.5$

Correct substitution and final ans-----1 ½ mark

SECTION D

Section D consists of 4 questions of 5 marks each

32.

Let the number of books purchased by the shopkeeper be x.

Cost price of x books = Rs 80

∴Original cost price of one book =Rs $\frac{80}{x}$ ---------------(1/2m)

If the shopkeeper had purchased 4 more books, then the number of books purchased by him would be (x + 4).

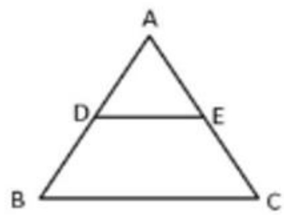
	$\therefore$New cost price of one book Rs = $\frac{80}{x+4}$ ---------------(1/2m) Given, Original cost price of one book − New cost price of one book = Rs 1 ---------------(1m) $\therefore \frac{80}{x} - \frac{80}{x+4} = 1$ ---------------(1/2m) $\Rightarrow \frac{80(x+4)-80x}{x(x+4)} = 1$ $\Rightarrow 80x + 320 - 80x = x(x+4)$ $\Rightarrow x^2 + 4x = 320$ ---------------(1m) $\Rightarrow x^2 + 4x = 320 = 0$ $\Rightarrow x^2 + 20x - 16x - 320 = 0$ $\Rightarrow x(x+20x) - 16(x+20) = 0$ $\Rightarrow x - 16 = 0$ or $x + 20 = 0$ $\Rightarrow x - 16 = 0$ or $x + 20 = 0$ $\Rightarrow x = 16$ Or $x = -20$ ---------------(1m) $\therefore x = 16$ ....($\because$ Number of books cannot be negative) Thus, the number of books purchased by the shopkeeper is 16. Rejecting negative value and accepting positive value-------(1/2m) **OR** Let us say that the train travels from A to B with a uniform speed s km/h and time taken by the train for the journey is t hours. The train travelled $d = 180\ km$ $\therefore\ t = \frac{180}{s}$ ....(1) ---------------(1m) On increasing its speed 9 km/h, it takes 1 hour less time to reach the same destination, as given in the question $t - 1 = \frac{180}{s+9}$ or ---------------(1m) $t = \frac{180}{s+9} + 1$ ....(2) on comparing (1) and (2) $t = \frac{180}{s} = \frac{180}{s+9} + 1$ ---------------(1m) Or $s(189 + s) = 180(s+9)$ $\Rightarrow s^2 + 189s = 180s + 1620$ $\Rightarrow s^2 + 9s - 1620 = 0$ Forming Q. Eqa---------------(1/2m) $\Rightarrow (s-36)(s+45) = 0$ Factorising ---------------(1m) If $s + 45 = 0$	

	$\Rightarrow s = -45$ $\because$ speed can't be negative,we reject this value. If $s - 36 = 0$ $\Rightarrow s = 36$ km/h $\therefore$ The speed of the train is 36 km/h.	Rejecting negative value and accepting positive value-------(1/2m)	

33. **Statement**:

If a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio. -------(1/2m)

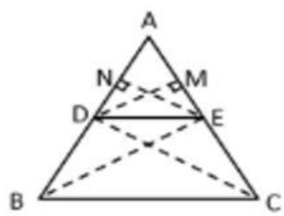
A ΔABC in which $DE \parallel BC$ and DE intersects AB and AC at D and E respectively.



To prove that:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Construction:



Join BE and CD.

Draw $EL \perp AB$ and $DM \perp AC$ ---(1/2m)

$ar(\Delta ADE) = \frac{1}{2} \times AD \times EL$ ---(1/2m)

$ar(\Delta ADE) = \frac{1}{2} \times AD \times EL$ ---(1/2m)

$ar(\Delta DBE) = \frac{1}{2} \times DB \times EL$

Therefore the ratio of these two is $\frac{ar(\Delta ADE)}{ar(\Delta DBE)} = \frac{AD}{DB}$.............. (1) ---(1/2m)

Similarly,

$ar(\Delta ADE) = ar(\Delta ADE) = \frac{1}{2} \times AE \times DM$

$ar(\Delta ECD) = \frac{1}{2} \times EC \times DM$

Therefore the ratio of these two is $\frac{ar(\Delta ADE)}{ar(\Delta ECD)} = \frac{AE}{EC}$.............. (2) ---(1/2m)

	Now, ΔDBE and ΔECD being on the same base DE and between the same parallels DE and BC, we have, $ar(\Delta DBE) = ar(\Delta ECD)$. . . . . (3) ---(1/2m) From equations 1, 2, 3 we can conclude that $\frac{AD}{DB} = \frac{AE}{EC}$ ---(1/2m) Hence we can say that the basic proportionality theorem is proved. In ΔOPE, AB || DE (Given) ∴ By Basic Proportionality Theorem, ---(1/2m) $\frac{OA}{AD} = \frac{OB}{BE}$ ...(i) Similarly, in ΔOEF, BC || EF (Given) ∴ $\frac{OB}{BE} = \frac{OC}{CF}$ ...(ii) ---(1/2m) Comparing (i) and (ii), we get $\frac{OA}{AD} = \frac{OC}{CF}$ Hence, AC || DF ---(1/2m) [By the converse of BPT]	

34. Let the height of the conical part be h.

Radius of the cone = Radius of the hemisphere = r = 21 cm

The toy can be diagrammatically represented as

Equ---(1/2m)

$$\text{Volume of the cone} = \frac{2}{3}\cdot \text{Volume of the hemisphere}$$

$$\therefore \frac{1}{3}\pi r^2 h = \frac{2}{3} \times \frac{2}{3}\pi r^3$$

Formula------(1 m)

$$\Rightarrow h = \frac{\frac{2}{3} \times \frac{2}{3}\pi r^3}{\frac{1}{3}\pi r^2}$$

$$\Rightarrow h = \frac{4}{3} r$$

$$\therefore h = \frac{4}{3} \times 21cm = 28cm$$

------------(1/2 m)

Thus, surface area of the toy = Curved surface area of cone + Curved surface area of hemispher

Equ---(1/2m)

$= \pi rl + 2\pi r^2$

$$= \pi r\sqrt{h^2 + r^2} + 2\pi r^2$$

Formula ----------(1 m)

$$= \pi r\left(\sqrt{h^2 + r^2} + 2r\right)$$

$$= \frac{22}{7} \times 21cm\left(\sqrt{(28cm)^2 + (21cm)^2} + 2 \times 21cm\right)$$

Finding "l"---(1 m)

$$= 66\left(\sqrt{784 + 441} + 42\right)cm^2$$

$$66\left(\sqrt{1225} + 42\right)cm^2$$

= 66(35+42) cm^2

= 66 x 77 cm^2

= 5082 cm^2

Correct answer--- (1/2m)

OR

Radius of the base of cylinder (r) = 2.8 m = Radius of the base of the cone (r)

Height of the cylinder (h)=3.5 m

Height of the cone (H)=2.1 m.

Slant height of conical part (l)=$\sqrt{r^2+H^2}$

= $\sqrt{(2.8)^2+(2.1)^2}$

= $\sqrt{7.84+4.41}$ 1

= $\sqrt{12.25}$ = 3.5 m 1

Area of canvas used to make tent = CSA of cylinder + CSA of cone 1

= 2×π×2.8×3.5 + π×2.8×3.5

= 61.6+30.8

= 92.4m^2 1

1

Cost of 1500 tents at ₹120 per sq.m

= 1500×120×92.4

= 16,632,000

Share of each school to set up the tents = 16632000/50 = ₹332,640

35. Median = 525, so Median Class = 500 – 600 ½

Class interval	Frequency	Cumulative Frequency
0–100	2	2
100–200	5	7
200–300	x	7+x
300–400	12	19+x
400–500	17	36+x
500–600	20	56+x
600–700	y	56+x+y
700–800	9	65+x +y
800–900	7	72+x+y
900–1000	4	76+x+y

1½

$76+x+y=100 \Rightarrow x+y=24$ ….(i) 1

Median = $l + \frac{\frac{n}{2}-cf}{f} \times h$ ½

Since, l=500, h=100, f=20, cf=36+x and n=100

Therefore, putting the value in the Median formula, we get;

$525 = 500 + \frac{50-(36+x)}{20} \times 100$ ½

so x = 9

y = 24 – x (from eq.i)

y = 24 – 9 = 15

Therefore, the value of x = 9 ½

and y = 15. ½

	SECTION E	

36. **Case study 1:**

(i)

$\sin 60° = \frac{PC}{PA}$ --------1/2m

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{18}{PA} \Rightarrow PA = 12\sqrt{3}$ m --------1/2 m

(ii)

$\sin 30° = \frac{PC}{PB}$ --------1/2m

$\Rightarrow \frac{1}{2} = \frac{18}{PB} \Rightarrow PB = 36$ m --------1/2 m

(iii)

$\tan 60° = \frac{PC}{AC} \Rightarrow \sqrt{3} = \frac{18}{AC}$

$\Rightarrow AC = 6\sqrt{3}$ m --------1/2 m

$\tan 30° = \frac{PC}{CB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{18}{CB}$ --------1/2m

$\Rightarrow CB = 18\sqrt{3}$ m ---------1/2m

Width AB = AC + CB = $6\sqrt{3} + 18\sqrt{3} = 24\sqrt{3}$ m --------1/2 m

or

RB = PC = 18 m &

PR = CB = $18\sqrt{3}$ m --------1/2 m

$\tan 30° = \frac{QR}{PR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{QR}{18\sqrt{3}}$ --------1/2m

$\Rightarrow QR = 18$ m ---------1/2m

QB = QR + RB = 18 + 18 = 36m. --------1/2 m

Hence height BQ is 36m

37.	**Case Study – 2** i) Since the production increases uniformly by a fixed number every year, the number of Cars manufactured in 1st, 2nd, 3rd, . . .,years will form an AP. So, a + 3d = 1800 & a + 7d = 2600 So d = 200 & a = 1200 ii) t_{12} = a + 11d ⇒ t_{12} = 3400 iii) $S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_{10} = \frac{10}{2}[2\,x\,1200 + (10-1)\,200]$ $\Rightarrow S_{10} = \frac{13}{2}[2\,x\,1200 + 9 \text{ x } 200]$ $\Rightarrow S_{10} =$ 5 x [2400 + 1800] $\Rightarrow S_{10} =$ 5 x 4200= 21000 [OR] Let in n years the production will reach to 31200 $S_n = \frac{n}{2}[2a + (n-1)d] = 31200 \Rightarrow \frac{n}{2}[2\,x\,1200 + (n-1)200] = 31200$ $\Rightarrow \frac{n}{2}[2 \text{ x } 1200 + (n-1)200] = 31200 \Rightarrow n\ [12 + (n-1)] = 312$ ⇒ n^2 + 11n -312 = 0 ⇒ n^2 + 24n - 13n -312 = 0 ⇒ (n +24)(n -13) = 0 ⇒ n = 13 or – 24. As n can't be negative. So n = 13	 ½ ½ ½ ½ ½ ½ ½ ½ ½ ½ ½ ½
38.	**Case Study 3** i) P(4,6), Q(3,2), R(6,5) ---------------plotting coordinates ½ m and three coordinates ½ mark each ii) P(-6,6) -------------------------plotting coordinates taking D as origin ½ m and finding coordinate of P , ½ m iii) Midpoint of PR=(5, 5.5)-----------------midpoint formula ½ m and correct answer 1½ m **[OR]** $PQ = \sqrt{(4-3)^2 + (6-2)^2} = \sqrt{17}$ ---------distance formula ½ m and correct answer 1 ½ m	